Optical Biosensors: Fundamentals in Optics A

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Content

- Light propagation description (rays, waves, polarization)
- Total internal reflection (TIR)
- Optical guided waves
- Definition of refractive index
- Elastic and inelastic scattering
- Light – matter manipulation, optical tweezers

Optics / Photonics - Light Propagation / Confinement

Propagation of light and its interaction with matter can be treated at different levels (accuracy):

Less accurate (and simpler to use…)

		Size of an object Δx
Ray optics	- refraction, reflection…	Δx>>λ
Wave optics	- wavelength *λ*, phase, interference…	Δx<~λ
Electromagnetic optics	- polarization, surface waves…	
Quantum optics	- quantized energies (photons), lasers…	

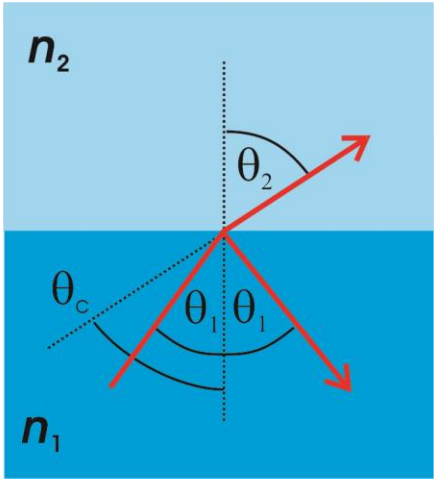
More general (and complicated…)

Ray Optics

Refractive index *n* describes optical density of matter in which a light beam – ray – propagates. At a plane interface between $n_1 > n_2$, reflection and refraction occurs.

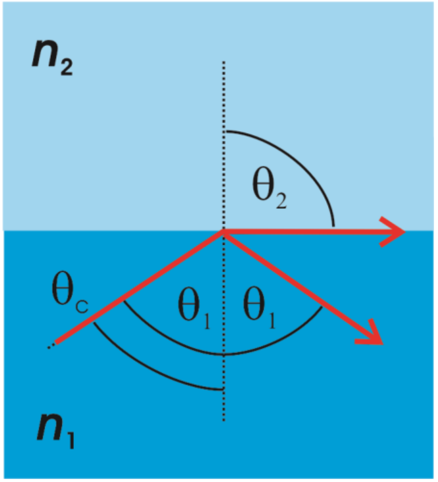
Snell law $n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \rightarrow \theta_c = \arcsin(n_2 / n_1)$

$\theta_1 < \theta_c$



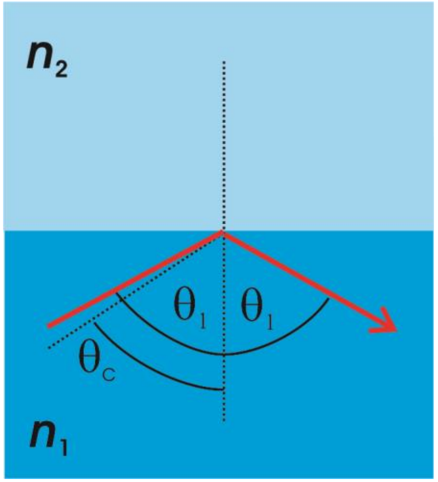
Refracted beam propagates to n_2 medium

$\theta_1 = \theta_c$



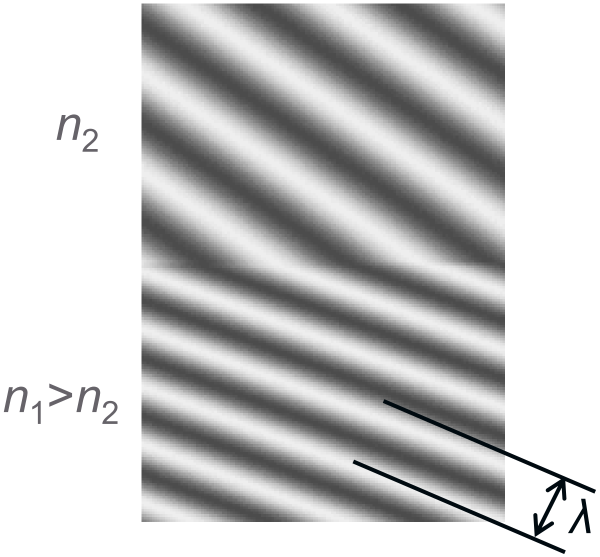
Beam propagates along the interface

$\theta_1 > \theta_c$



Beam undergoes total internal reflection - TIR

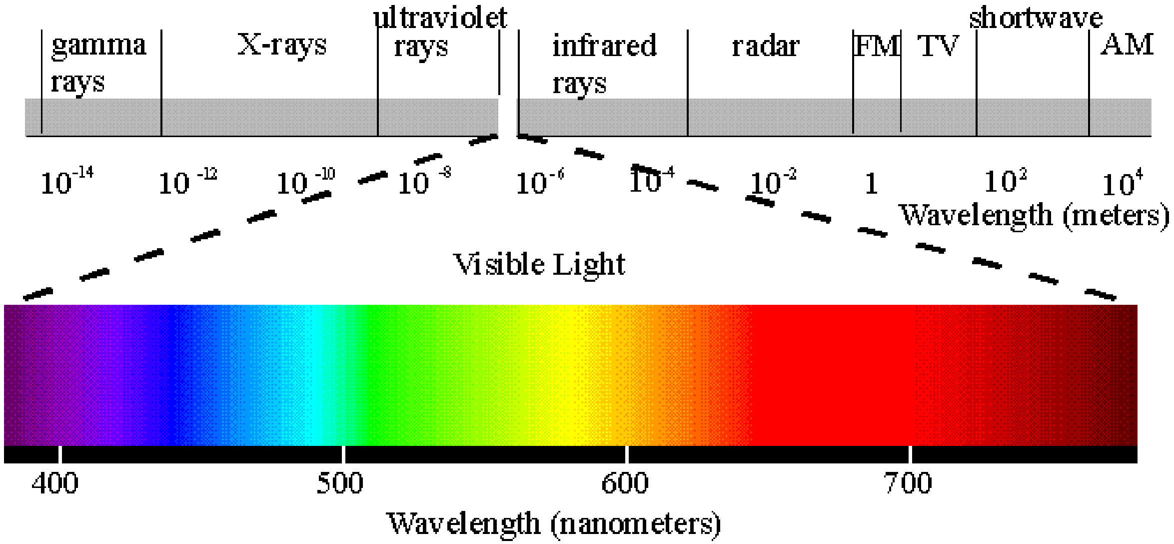
Wave Optics



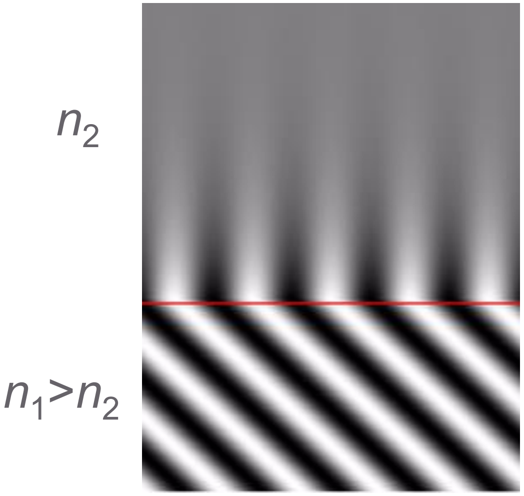
Wave equation $$\left(\frac{d^2}{dx^2}-\frac{1}{v^2}\frac{d^2}{dt^2}\right)u(x,t)=0$$

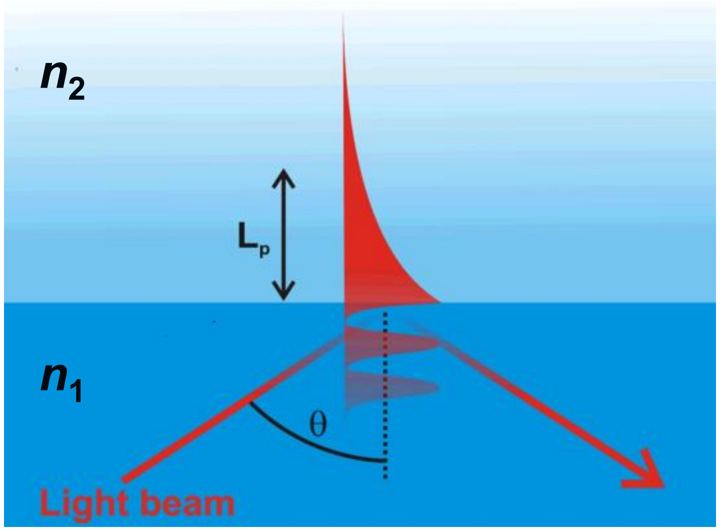
Plane wave (scalar) $$u(x,t)=A\sin\left(k_0 nx-\omega t\right)$$

$k_0=\omega/c$	propagation constant in vacuum
ω	angular frequency
$v=c/n$	velocity of light
$\lambda=2\pi c/\omega$	wavelength in vacuum



Total Internal Reflection - TIR





At angles $\theta>\theta_c$ field amplitude exponentially decays to medium n_2

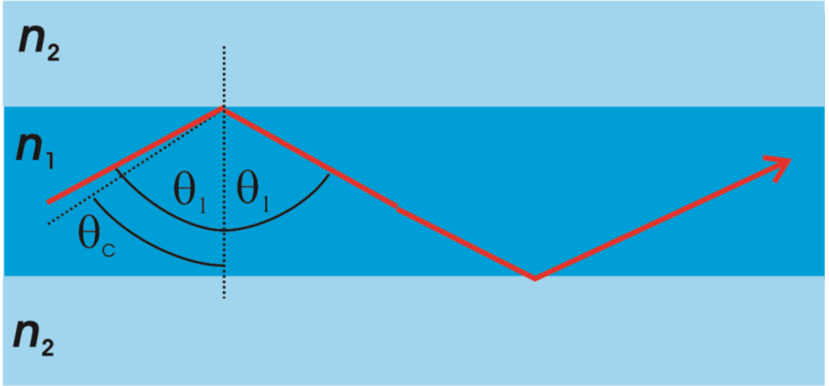
$$u(x)=u_0 e^{-x/L_p}$$

The field probes limited penetration depth from the interface:

$$L_p=\frac{\lambda}{4\pi\sqrt{n_1^2\sin^2(\theta)-n_2^2}}$$

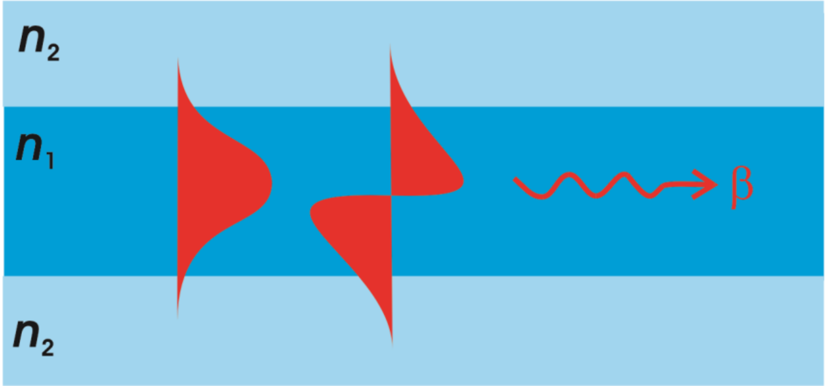
Slab Optical Waveguide

Ray optics point of view:



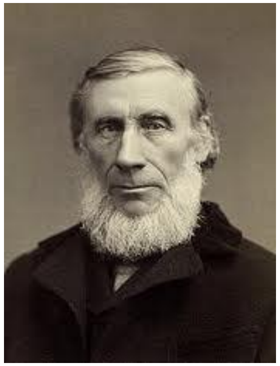
Light propagation is confined by TIR at opposite interfaces

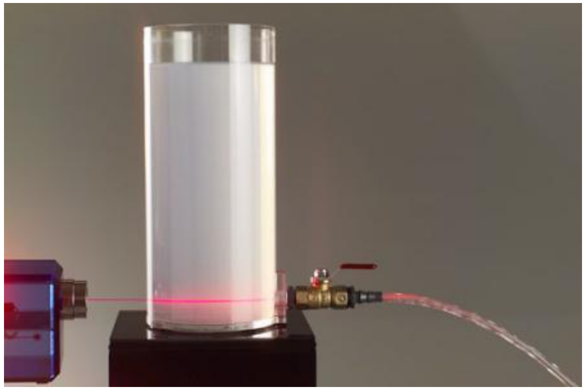
Wave optics point of view:



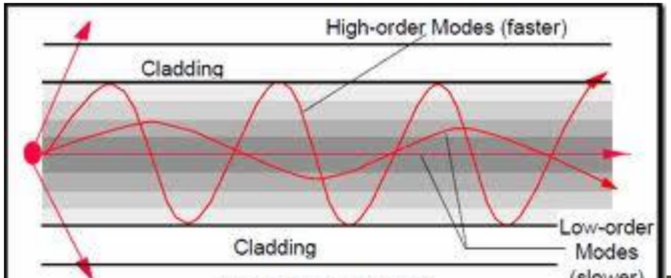
Only discrete modes with certain propagation constants β can travel through the waveguide

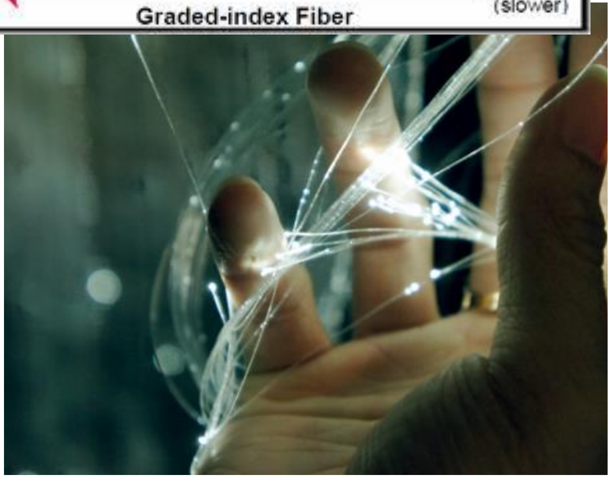
Examples of Dielectric Optical Waveguide



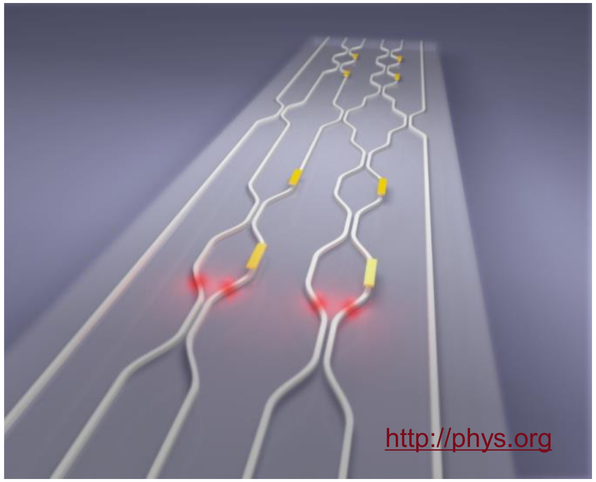


Historical Tyndall experiment at 1870.





Optical fibers



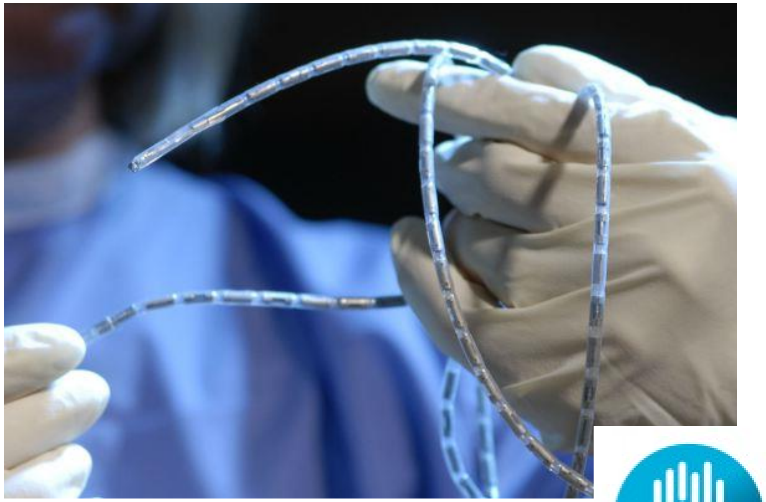
http://phys.org

Optical circuits

Examples of Dielectric Optical Waveguide

http://www.schott.com

Optical fiber bundles used for imaging of remote places (right) and catheters (down).



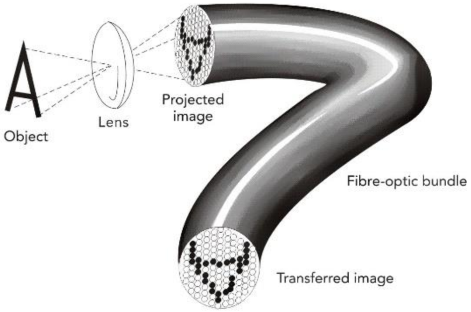
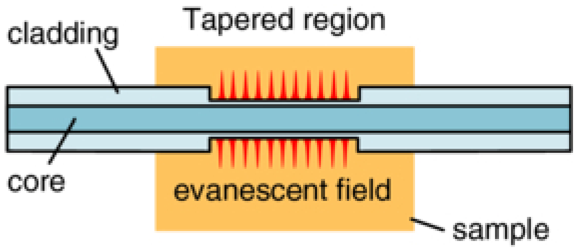


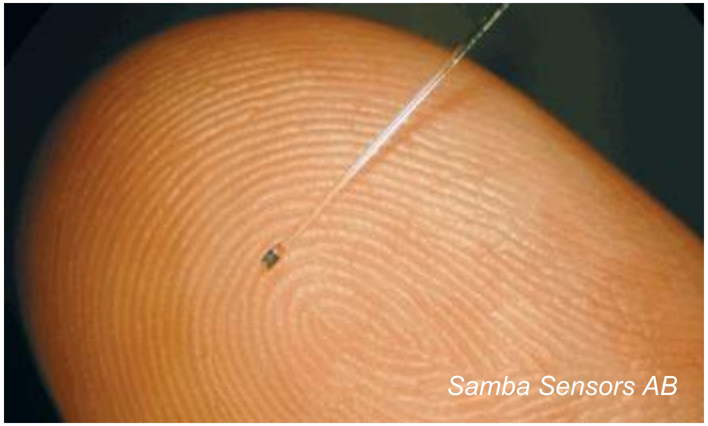
Figure 7-4 Fiber optic endoscopy

Examples of Dielectric Optical Waveguide

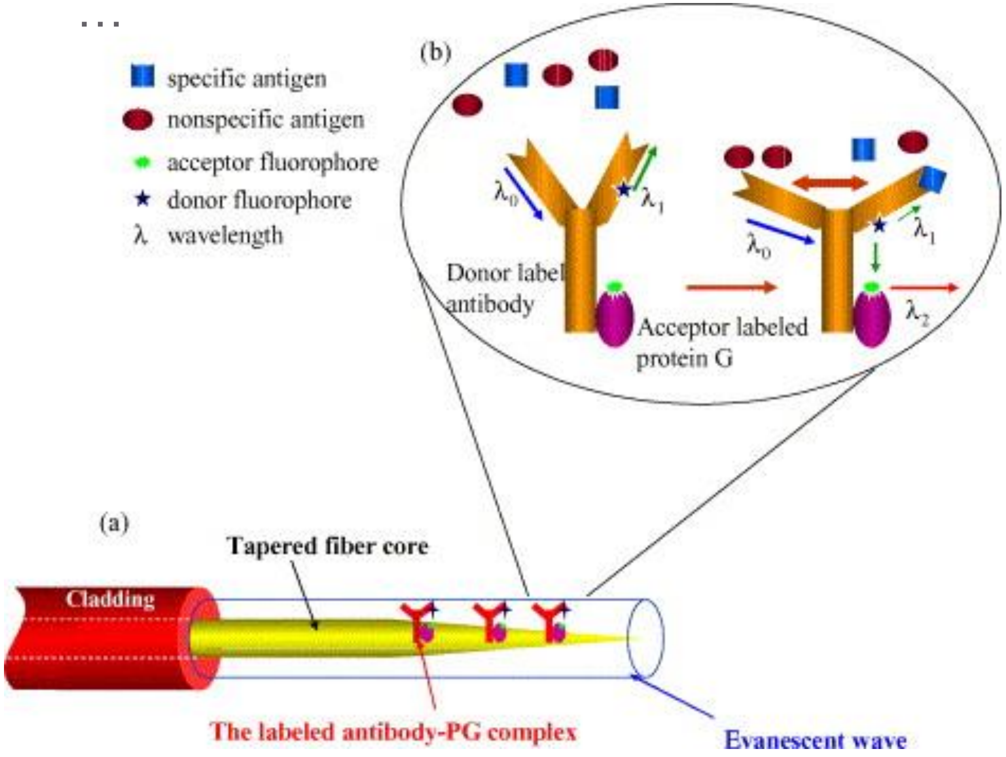
Optical fibers allow for design of miniature sensors relying on probing by evanescent field:



Hodgkinson et al, 2013 *Meas. Sci. Technol.* **24** 012004


Samba Sensors AB

Oxygen
Pressure
Fluorescence, FRET
…



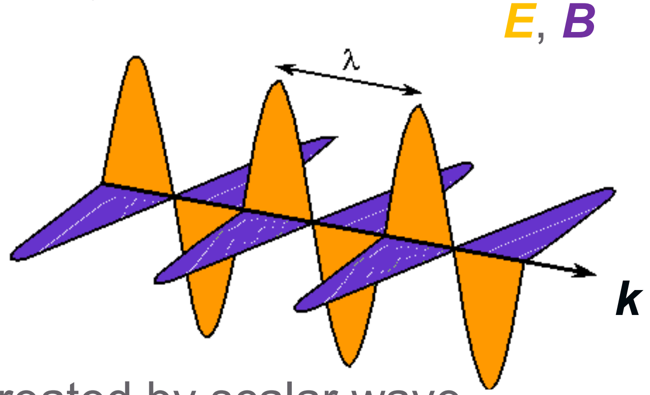
Sungho Biosensors and Bioelectronics
Volume 21, Issue 7, 15 January 2006, Pages 1283–1290

Electromagnetic Optics

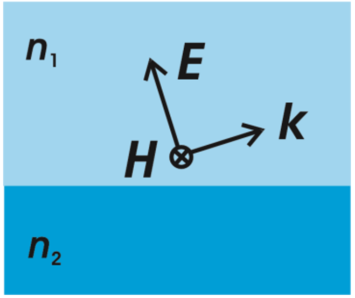
Describes light as a vector field with electric *E* and magnetic *B* components. Polarization (orientation of *E* and *B* vectors) is important in additional phenomena (e.g. surface plasmon resonance - SPR). Solution can be determined by set of Maxwell equations and boundary conditions:

$$\nabla \times \vec{E} + \frac{\partial}{\partial t}\vec{B} = 0 \qquad \nabla \times \vec{H} - \frac{\partial}{\partial t}\vec{D} = \vec{j}$$

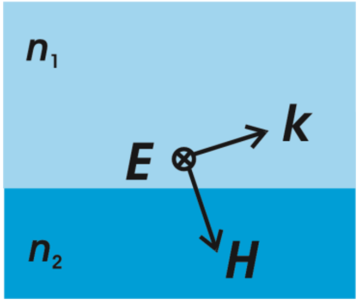
$$\nabla . \vec{D} = \rho \qquad \nabla . \vec{B} = 0$$



For planar interfaces, two set of solution can be treated by scalar wave equation:



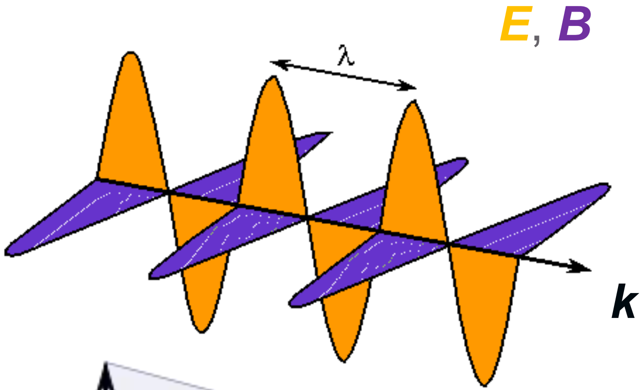
Transverse magnetic (TM, p)



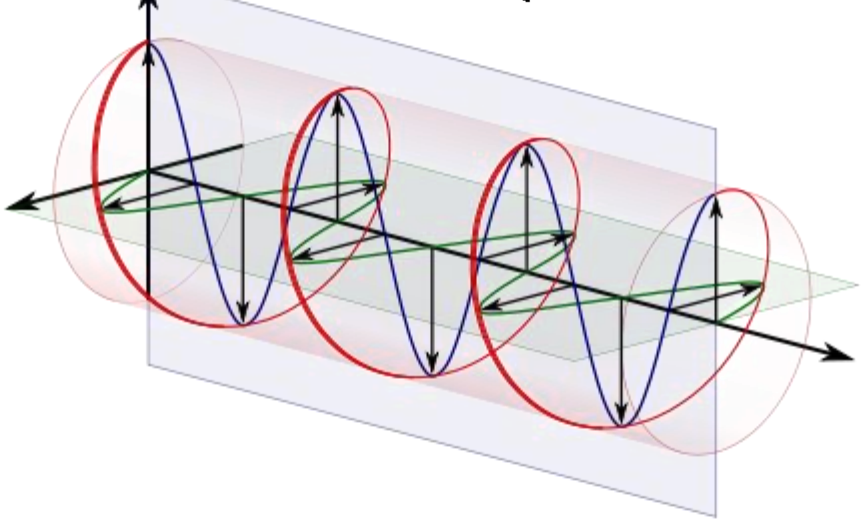
Transverse electric (TE, s)

Electromagnetic Optics
Polarization

Linear polarization:



Circular polarization: can be described as a combination of two phase shifted linear polarizations.



'Maxwell‘s Macroscopic Equations'

The interaction of EM waves with matter is introduced through the permittivity *ε* and permeability *μ*. Attributed to space filled with atoms / molecules… and averaged over their ensemble (thus macroscopic, size << *λ* and >> then distance between atoms).

$\vec{E}$	Electric field	$\vec{j}$	Current density
$\vec{B}$	Magnetic vector	ρ	Charge density
$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$	Displacement field	$\vec{P}$	Polarizability
$\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$	Magnetizing field	$\vec{M}$	Magnetization
$c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$	Light velocity in vacuum		

Dielectric Materials

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E}\left(1+\chi\right) = \varepsilon_0 \varepsilon \vec{E}$$

χ Susceptibility

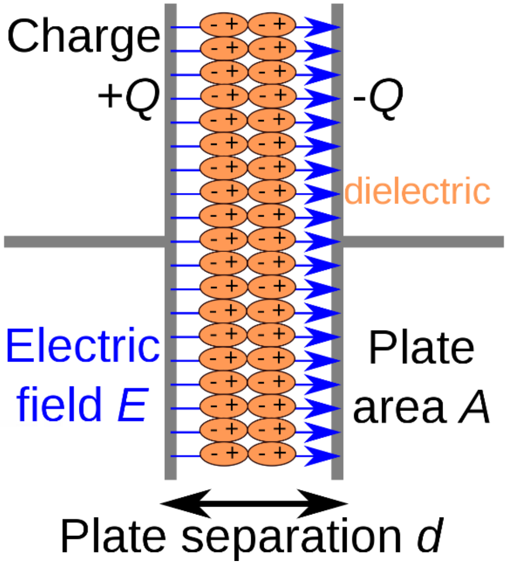
For more details, see Lorenz Lorenz or Clausius-Mossotti theories for taking the local and macroscopic field into account:

$$\frac{\chi}{\left(3+\chi\right)} = \frac{N\alpha}{3\varepsilon_0}$$

α Polarizability

$$\boxed{n^2 = \varepsilon}$$

Refractive index



$$n^2 + 2 = \frac{3}{1 - \frac{4}{3}\pi \frac{N_A}{M_w} \rho\alpha}$$

n – Refractive Index
N_A – Avogadro's constant
M_w – Molar weight
ρ – Density
α – Molecular polarizability

Yang; C., Jenekhe, S. *Chem. Mater.* 1995, *7*, 1276

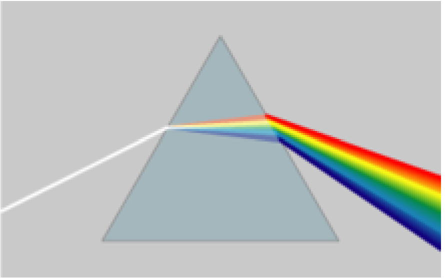
Complex Refractive Index

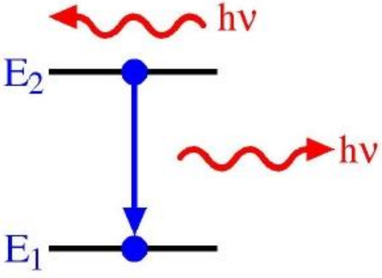
Parameter that describes interaction of light with matter composed of elements (e.g. atoms) that are << *λ* and exhibit polarizability. By averaging over many atoms that are be polarized by the oscillating electric field.

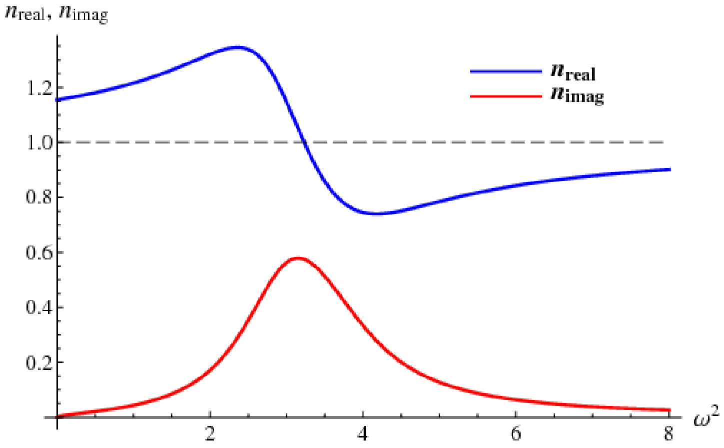
$$\tilde{n} = n + ik$$

Real part – refraction

Imaginary part - absorption

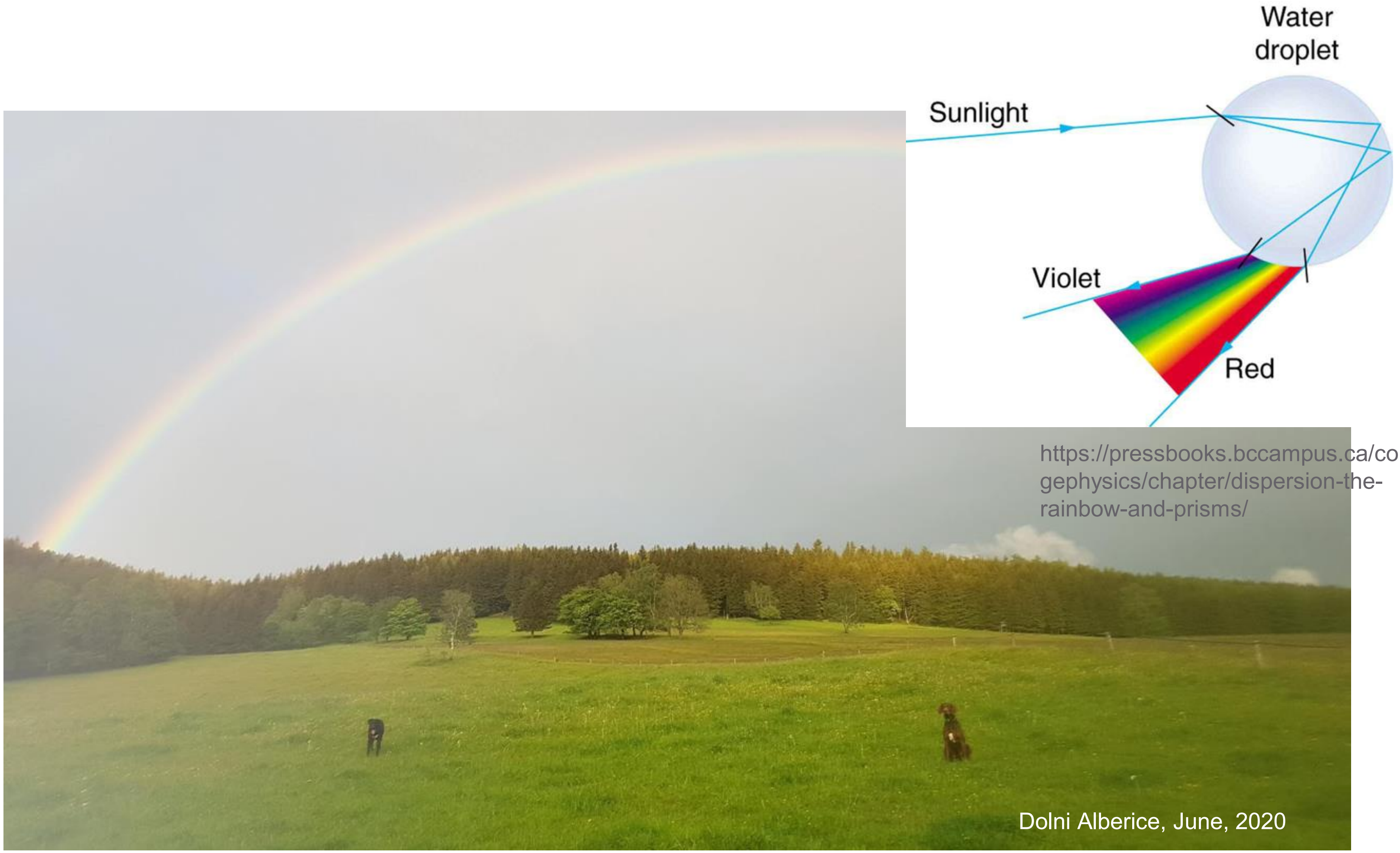






Kramers-Kronig relations

Dispersion of Refractive Index

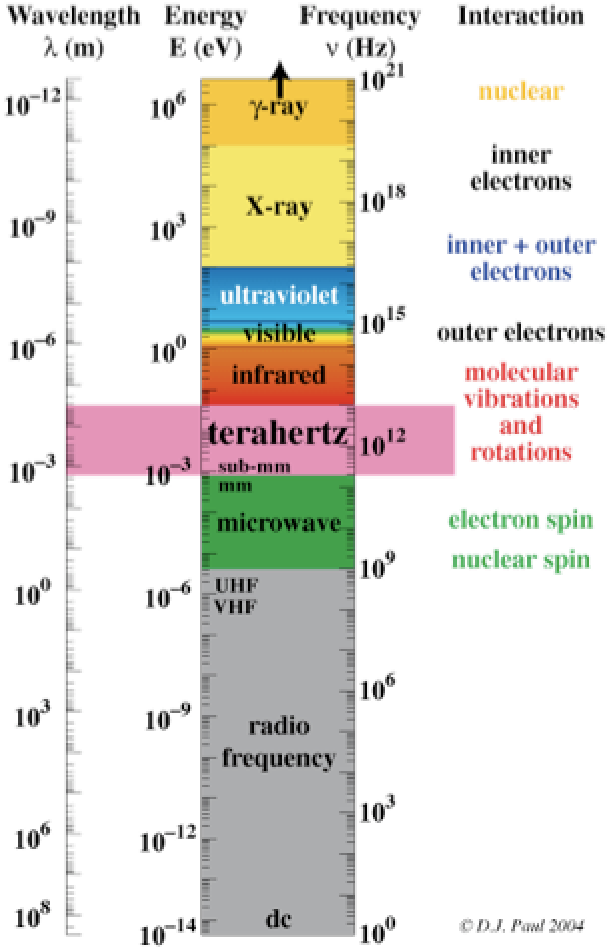


https://pressbooks.bccampus.ca/collegephysics/chapter/dispersion-the-rainbow-and-prisms/

Dolni Alberice, June, 2020

Absorption of Molecules

Absorption of light by molecules is accompanied with their transition from a ground state to excited state (followed by a relaxation). It typically occurs at distinct energies leading to specific bands in absorption spectrum:



Electronic lines correspond to a change in the electronic state of an atom or molecule. Typically UV-Vis.

Vibrational lines correspond to changes in the vibrational state of the molecule and are typically found in the infrared region.

Rotational lines, for instance, occur when the rotational state of a molecule is changed. Rotational lines are typically found in the microwave spectral region.

Combination of above can lead to rather complex spectra.

Complex Refractive Index

Often electric field is described by using complex numbers, reason is that mathematically some operations can be performed easier. However, physical meaning has only the real (or imaginary) part of it.

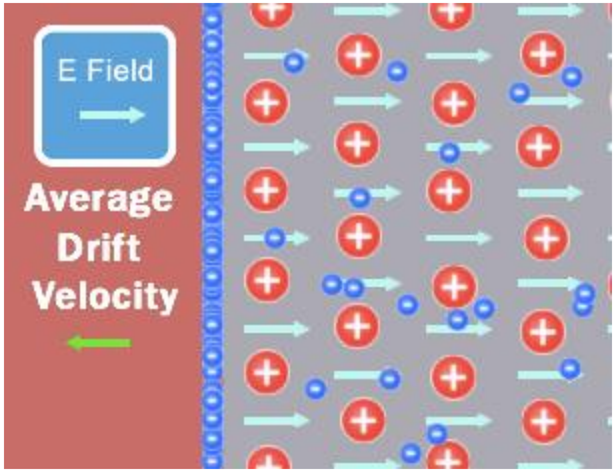
$$\vec{E} = \mathrm{Re}\left\{ \vec{E}_0 e^{-i\left(\omega t - \frac{2\pi}{\lambda}[n+ik]x\right)} \right\} = \vec{E}_0 \sin\left(\omega t - \frac{2\pi}{\lambda} nx\right) e^{-\frac{2\pi}{\lambda}kx}$$

Wave behavior (as discussed for scalar field)

Exponential decay, e.g. related Beer–Lambert law

Metallic Materials

Refractive index of metals can be described by Drude model. It deals with electron density cloud that is fluid and a static positively charged lattice representing the metal crystal lattice.



$$n^2 = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$$

$$\omega_p^2 = \frac{Ne^2}{m_e \varepsilon_0}$$ Plasma frequency

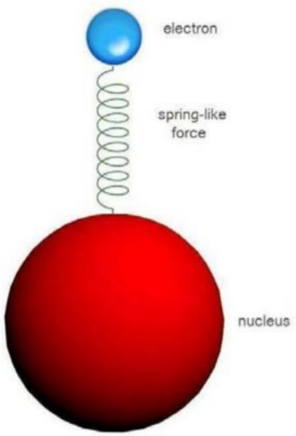
For $\omega < \omega_p$ – n is complex and radiation is attenuated.
For $\omega > \omega_p$ – n is real and radiation is not attenuated(transparent).

ω_p for Au, Ag is of 9.6, 8.5 eV and for visible wavelengths behaves as metallic (highly reflective non-transparent).

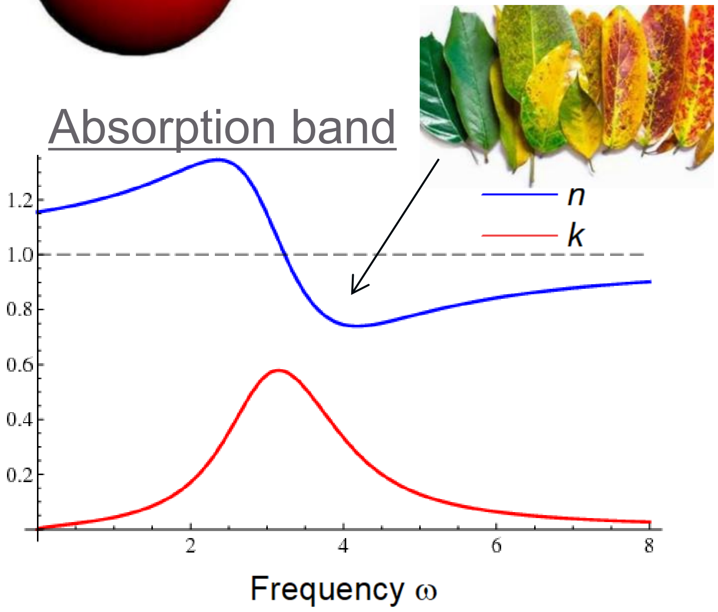
Dielectrics and Metals

Refractive index – describes how electromagnetic field couples with matter, described as a complex number: *n*+i·*k*.

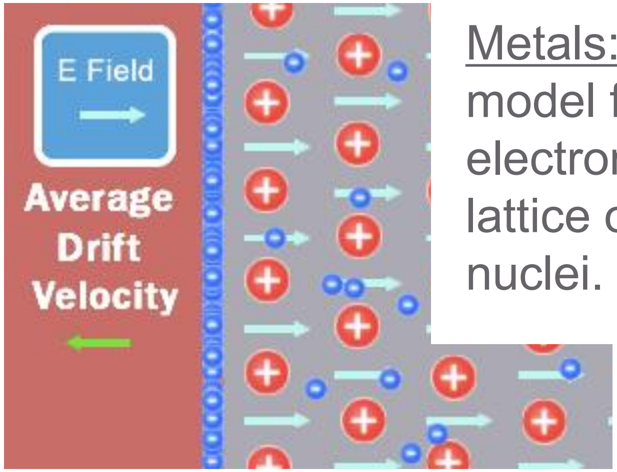
Dielectrics: oscillator model for tightly bound electrons to atom nuclei.



Absorption band

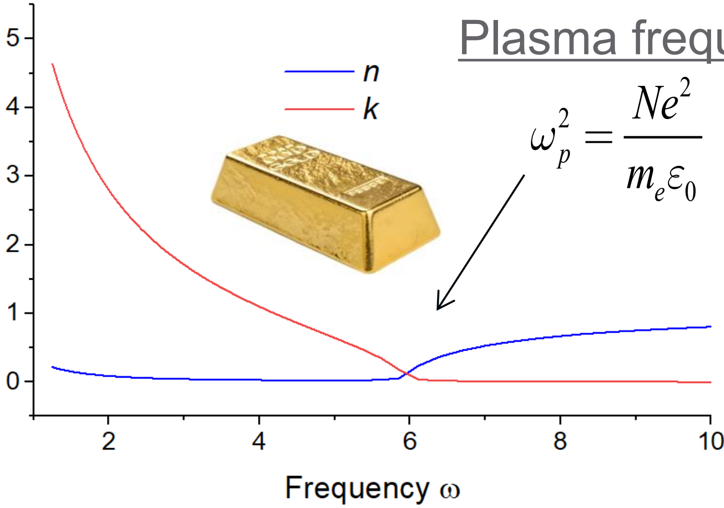


Metals: Drude model for free electrons in a lattice of atom nuclei.



Plasma frequency

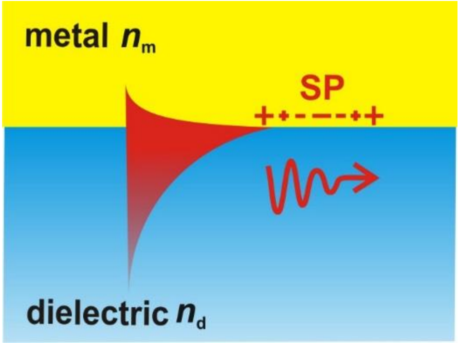
$$\omega_p^2 = \frac{Ne^2}{m_e \varepsilon_0}$$



Metallic Waveguides – Surface Plasmons (SP)

Surface plasmons (SPs) or also called surface plasmon polaritons (SPPs) are waves originating from coupled oscillations of electron plasma density and associated electromagnetic field on a metal – dielectric interface.

They travel along single interface which serves a waveguide.



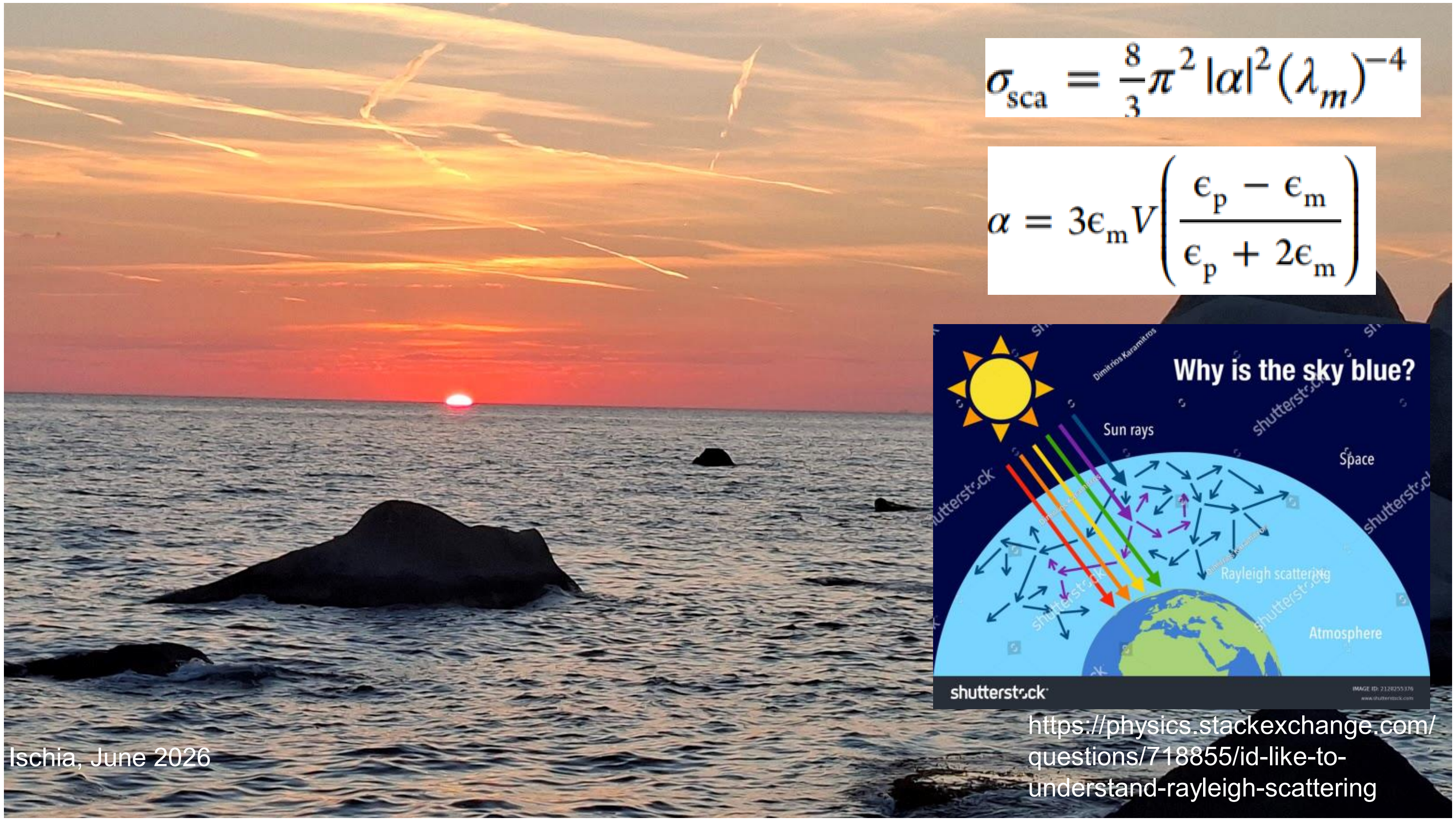
Propagation constant β can be analytically expressed as:

$$\beta = \frac{2\pi}{\lambda}\sqrt{\frac{n_m^2 n_d^2}{n_m^2 + n_d^2}}$$

- SPs allows for tight confinement of electromagnetic field at the interface.
- For visible near infrared wavelength typically gold and silver is used where the Re{n_m^2}<0.
- Majority of the field is probing the dielectric n_d.

Elastic scattering

Energy of photons / wavelength does not change. Important example of elastic scattering is Rayleigh scattering.



$$\sigma_{\mathrm{sca}} = \frac{8}{3}\pi^2 |\alpha|^2 (\lambda_m)^{-4}$$

$$\alpha = 3\epsilon_{\mathrm{m}} V \left(\frac{\epsilon_{\mathrm{p}} - \epsilon_{\mathrm{m}}}{\epsilon_{\mathrm{p}} + 2\epsilon_{\mathrm{m}}} \right)$$

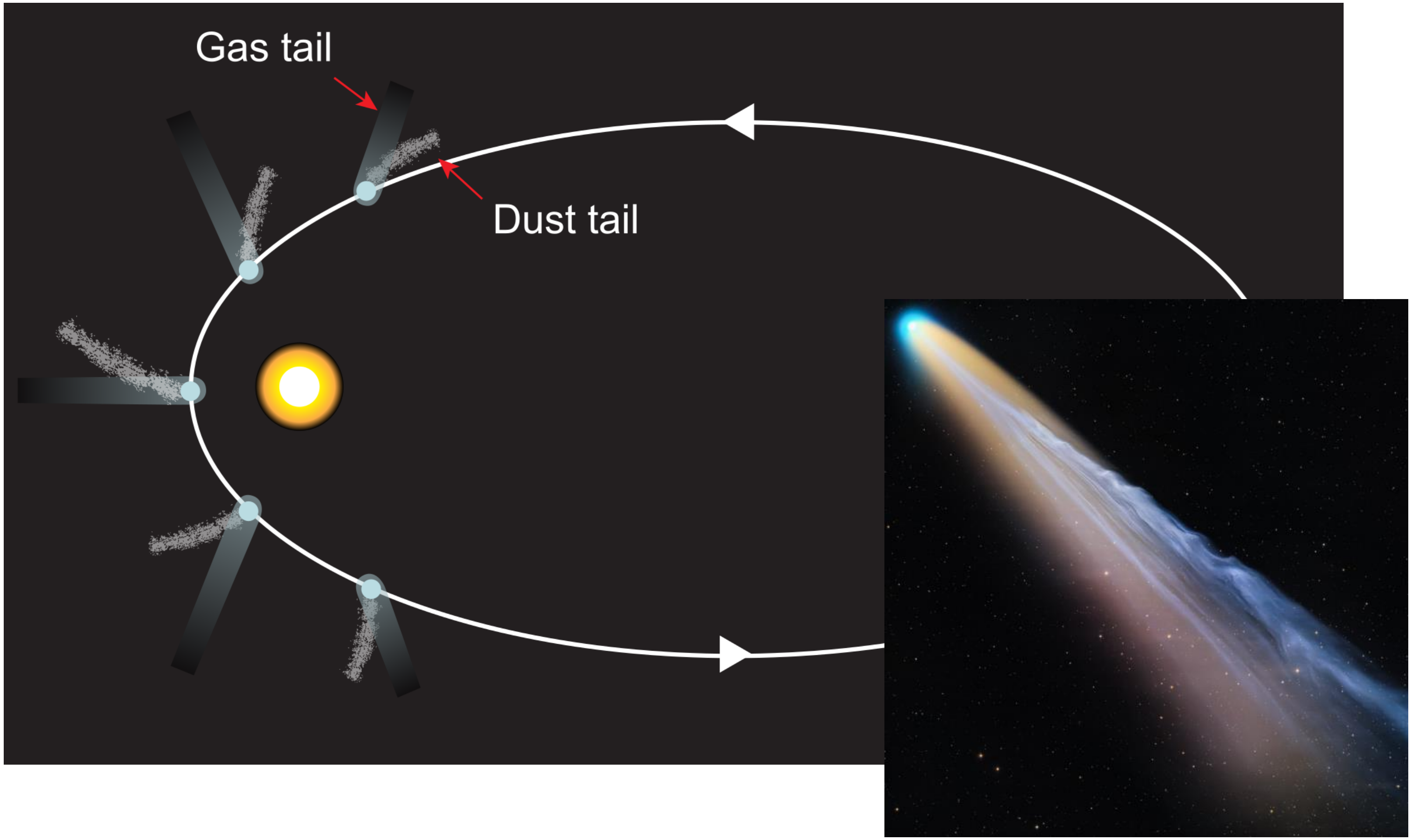
Ischia, June 2026

https://physics.stackexchange.com/questions/718855/id-like-to-understand-rayleigh-scattering

“Optical Tweezers”

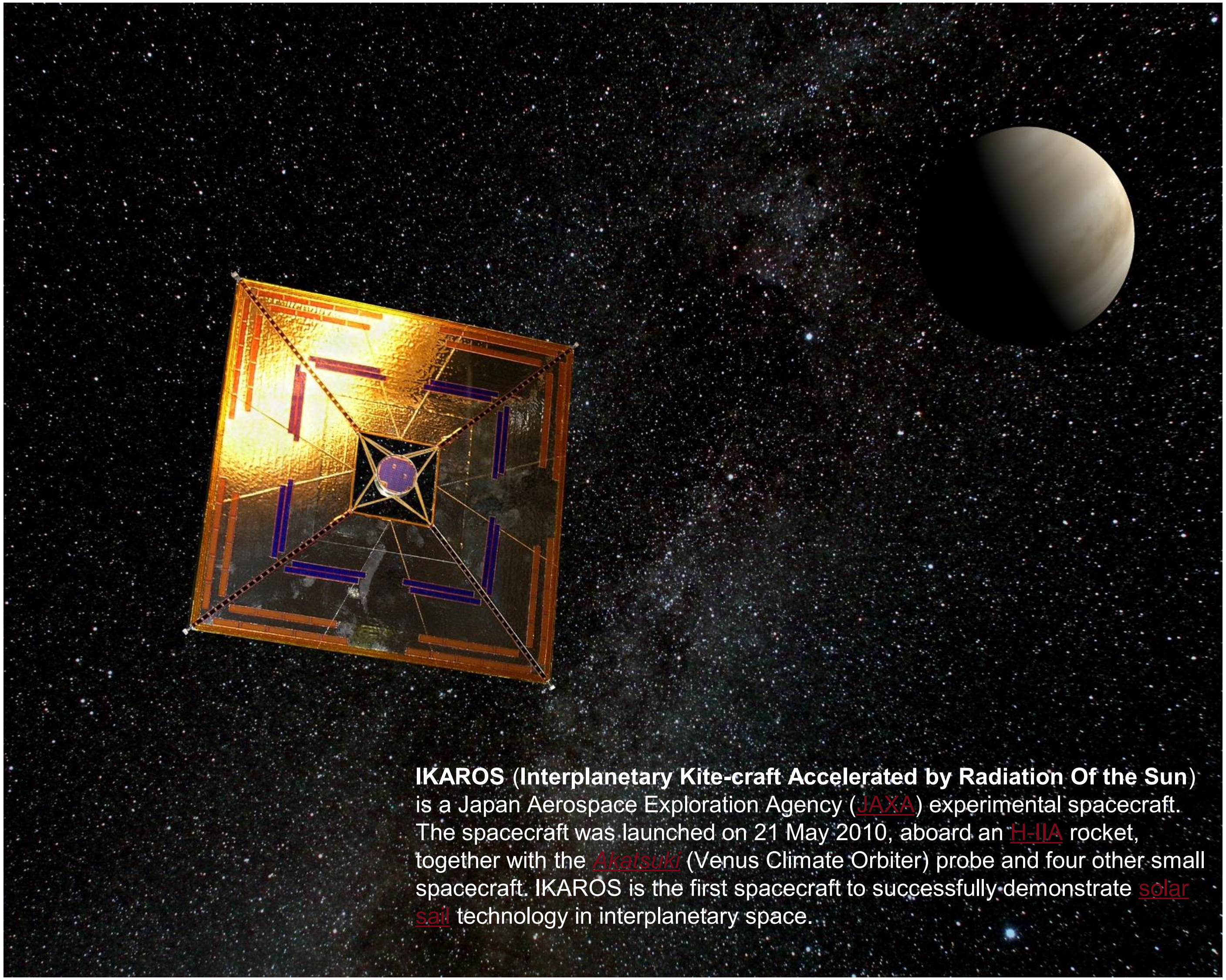
The Nobel Prize in Physics 2018 was awarded to Arthur Ashkin, Gérard Mourou and Donna Strickland. Their inventions have revolutionised laser physics. Extremely small objects and incredibly rapid processes are now being seen in a new light. Advanced precision instruments are opening up unexplored areas of research and a multitude of industrial and medical applications.

Comet tail



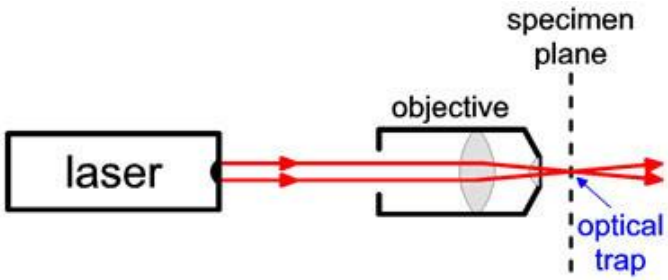
https://en.wikipedia.org/wiki/Comet_tail

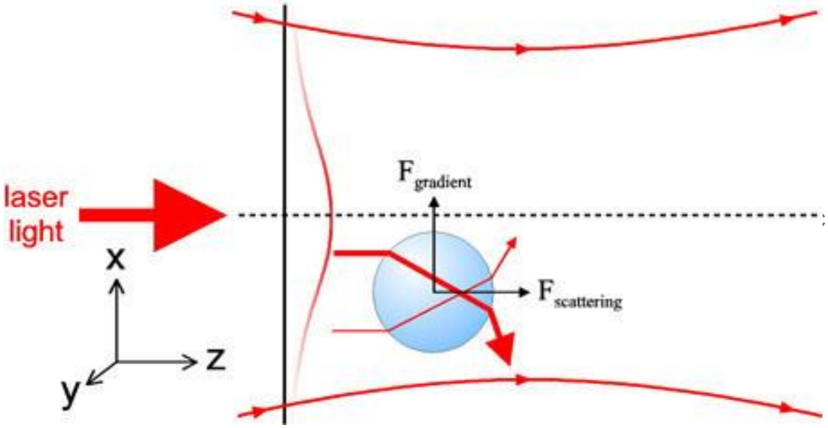
Solar Sail



IKAROS (**Interplanetary Kite-craft Accelerated by Radiation Of the Sun**) is a Japan Aerospace Exploration Agency (JAXA) experimental spacecraft. The spacecraft was launched on 21 May 2010, aboard an H-IIA rocket, together with the *Akatsuki* (Venus Climate Orbiter) probe and four other small spacecraft. IKAROS is the first spacecraft to successfully demonstrate solar sail technology in interplanetary space.

Confinement of Light Creates Optical Trap





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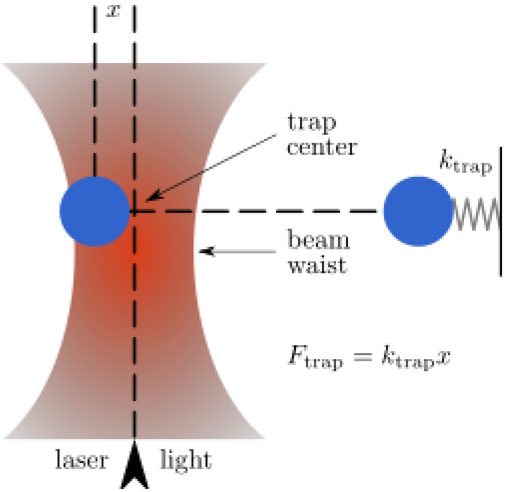
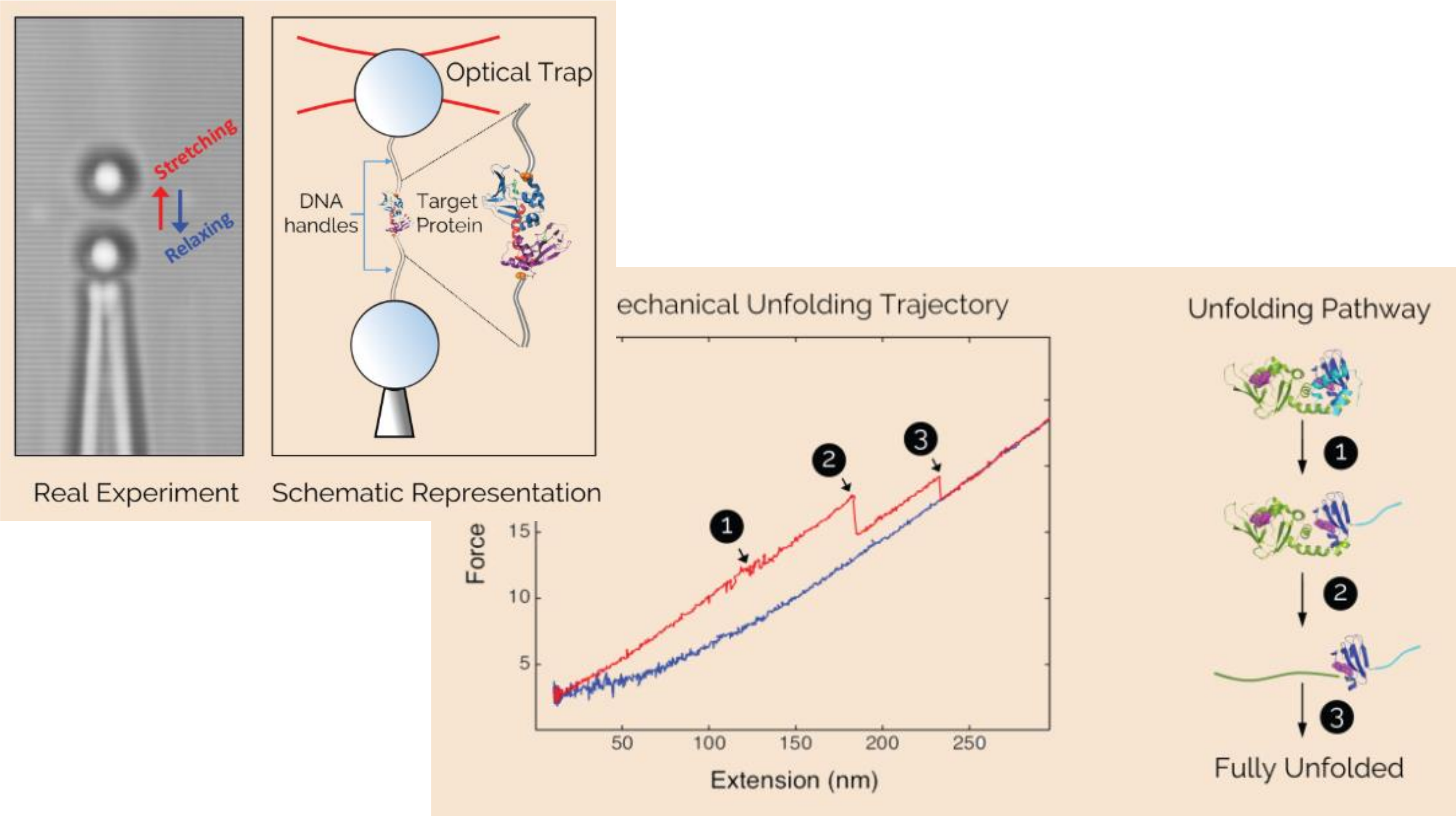
https://blocklab.stanford.edu/optical_tweezers.html



https://en.wikipedia.org/wiki/Optical_tweezers

Ashkin, A. (1970). "Acceleration and Trapping of Particles by Radiation Pressure". *Phys. Rev. Lett*. **24** (4): 156–159.

Force Measurements



http://maillardlab.org/optical-tweezers/